Closing Tue: 13.3(2)
Closing Thu: 13.4, 14.1
Midterm 1 will be returned Tuesday.

Summary of TNB-Frame Facts:

Given $\boldsymbol{r}(t) = < x(t), y(t), z(t) >$

$$\boldsymbol{T}(t) = \frac{\boldsymbol{r}'(t)}{|\boldsymbol{r}'(t)|} = \text{unit tangent}$$

$$\boldsymbol{N}(t) = \frac{\boldsymbol{T}'(t)}{|\boldsymbol{T}'(t)|} = \text{principal unit normal}$$

$$\boldsymbol{B}(t) = \boldsymbol{T}(t) \times \boldsymbol{N}(t) = \text{binormal}$$

Normal Plane: Plane thru a point on the curve and orthogonal to any tangent vector at that point.

Osculating Plane:
Plane thru a point on the curve and parallel to both $\boldsymbol{r}'(t)$ and $\boldsymbol{r}''(t)$.

Entry Task:

$$\boldsymbol{r}(t) = < 2\sin(3t), t, 2\cos(3t) >$$

Compute

1. $\boldsymbol{r}'(t)$
2. $\boldsymbol{r}''(t)$
3. $\boldsymbol{T}(t)$
4. $\boldsymbol{N}(t)$
5. $\boldsymbol{B}(t)$
6. At $t = \pi$, give the tangent line.
7. At $t = \pi$, give the normal plane.
8. At $t = \pi$, give the oscullating plane.

TNB Notes:

- ***T***, ***N***, and ***B*** always have length one.

- The **tangent line** is:
  (a) parallel to $\boldsymbol{r}'(t)$ and $\boldsymbol{T}(t)$.
  (b) orthogonal to $\boldsymbol{N}(t)$ and $\boldsymbol{B}(t)$.

- The **normal plane** is:
  (a) parallel to $\boldsymbol{N}(t)$ and $\boldsymbol{B}(t)$.
  (b) orthogonal to $\boldsymbol{r}'(t)$ and $\boldsymbol{T}(t)$.

- The **osculating** (kissing) **plane** is:
  (a) parallel to $\boldsymbol{T}(t)$, $\boldsymbol{r}'(t)$, $\boldsymbol{N}(t)$, & $\boldsymbol{r}''(t)$.
  (b) orthogonal to $\boldsymbol{B}(t)$ and $\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)$.

13.4 Position, Velocity, Acceleration

If *t = time* and position is given by

$$\boldsymbol{r}(t) = < x(t), y(t), z(t) >$$

then

$$\boldsymbol{r}'(t) = \lim_{h\to 0} \frac{\boldsymbol{r}(t+h) - \boldsymbol{r}(t)}{h}$$
$$= \frac{\text{change in position}}{\text{change in time}}$$
$$= \text{velocity} = \boldsymbol{v}(t)$$

$$|\boldsymbol{r}'(\boldsymbol{t})| = \frac{\text{change in dist}}{\text{change in time}} = \text{speed}$$

and

$$\boldsymbol{r}''(t) = \lim_{h\to 0} \frac{\boldsymbol{r}'(t+h) - \boldsymbol{r}'(t)}{h}$$
$$= \frac{\text{change in velocity}}{\text{change in time}}$$
$$= \text{acceleration} = \boldsymbol{a}(t)$$

Let t be **time in seconds** and assume the position of an object (in **feet**) is given by

$$\boldsymbol{r}(t) = < t, 2e^{-t}, 0 >$$

Compute

1. $\boldsymbol{r}'(t)$ and $\boldsymbol{r}''(t)$
2. $\boldsymbol{r}'(0)$ and $\boldsymbol{r}''(0)$
3. Give the oscullating plane at *t = 0*. (no work needed)

HUGE application:
Modeling ANY motion problem.

Newton's 2^{nd} Law of Motion states

Force = mass · acceleration

$$\boldsymbol{F} = m \cdot \boldsymbol{a}$$

If $\boldsymbol{F} = \langle 0,0,0 \rangle$, then all the forces 'balance out' and the object has no acceleration. (Velocity will remain constant)

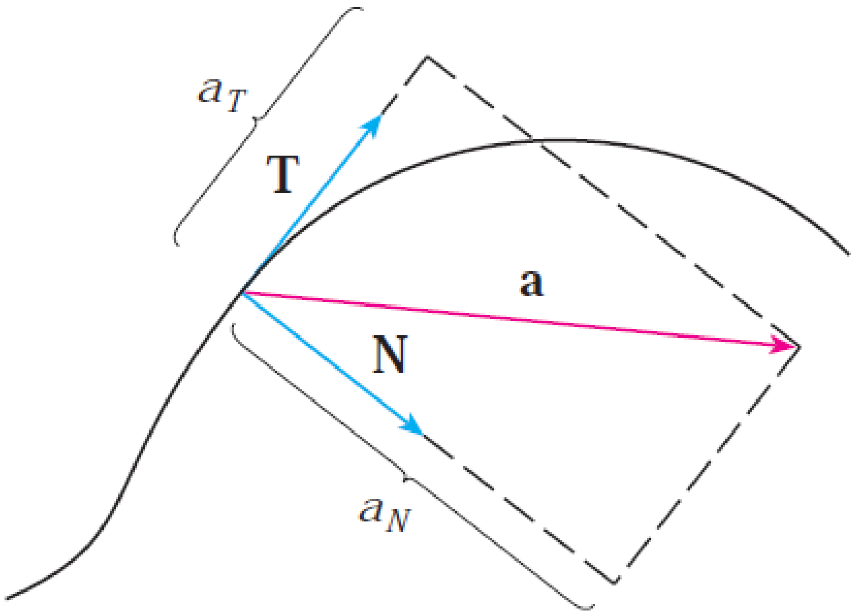
If $\boldsymbol{F} \neq \langle 0,0,0 \rangle$, then acceleration will occur, and we integrate to find velocity and position.

Example:
A ball with mass *m = 0.8* kg is thrown northward into the air with initial speed of 30 m/sec at an angle of 30 degrees with the ground.
A west wind applies a steady force of 4 N on the ball (west to east).
If you are standing on level ground, where does the ball land?

Steps (for *all* motion problems):

1. Forces?
2. Get acceleration.
3. Integrate to get $\vec{\boldsymbol{v}}(t)$
   (initial conditions?)
4. Integrate again to get $\vec{\boldsymbol{r}}(t)$
   (initial conditions?)

Measuring and describing acceleration



Recall: $\mathrm{comp}_{\boldsymbol{b}}(\boldsymbol{a}) = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{\boldsymbol{b}}$ = length.

We define the tangential and normal components of acceleration by:

$a_T = \mathrm{comp}_{\boldsymbol{T}}(\boldsymbol{a}) = \boldsymbol{a}\cdot\boldsymbol{T}$ = tangential

$a_N = \mathrm{comp}_{\boldsymbol{N}}(\boldsymbol{a}) = \boldsymbol{a}\cdot\boldsymbol{N}$ = normal

Note that: $\boldsymbol{a} = a_T\boldsymbol{T} + a_N\boldsymbol{N}$

Derivation of interpretation:

Let $\nu(t) = |\vec{\boldsymbol{v}}(t)|$ = speed.

1. $\vec{\boldsymbol{T}}(t) = \frac{\vec{\boldsymbol{r}}'(t)}{|\vec{\boldsymbol{r}}'(t)|} = \frac{\vec{\boldsymbol{v}}(t)}{\nu(t)}$ implies $\vec{\boldsymbol{v}} = \nu\vec{\boldsymbol{T}}$.

2. $\kappa(t) = \frac{|\vec{\boldsymbol{T}}'(t)|}{|\vec{\boldsymbol{r}}'(t)|} = \frac{|\vec{\boldsymbol{T}}'|}{\nu(t)}$ implies $|\vec{\boldsymbol{T}}'| = \kappa\nu$.

3. $\vec{\boldsymbol{N}}(t) = \frac{\vec{\boldsymbol{T}}'(t)}{|\vec{\boldsymbol{T}}'(t)|} = \frac{\vec{\boldsymbol{T}}'}{\kappa\nu}$,implies $\vec{\boldsymbol{T}}' = \kappa\nu\vec{\boldsymbol{N}}$.

Differentiating the first fact above gives

$$\vec{\boldsymbol{a}} = \vec{\boldsymbol{v}}' = \nu'\vec{\boldsymbol{T}} + \nu\vec{\boldsymbol{T}}', \text{ so}$$

$$\vec{\boldsymbol{a}} = \vec{\boldsymbol{v}}' = \nu'\vec{\boldsymbol{T}} + k\nu^2\vec{\boldsymbol{N}}.$$

Conclusion

$a_T = \nu' = \frac{d}{dt}|r'(t)| =$ “deriv. of speed”

$a_N = k\nu^2 =$ curvature · (speed)^2

For computational purposes, we use

$$a_T = \frac{\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{r}}''}{|\vec{\boldsymbol{r}}'|} \quad \text{and} \quad a_T = \frac{|\vec{\boldsymbol{r}}'\times\vec{\boldsymbol{r}}''|}{|\vec{\boldsymbol{r}}'|}$$

Example:

$$\vec{\boldsymbol{r}}(t) = < \cos(t) \, , \sin(t) \, , t >$$

Find the tangential and normal components of acceleration.